

## Preliminary Planning Sheet Grade 7 – Billy's Goat

### Domain(s)

Geometry

### Standard(s)

7.G.B.4

### Mathematical Practices

MP.1 MP.2 MP.3 MP.4 MP.5 MP.6

### Major Underlying Mathematical Concepts

- Area
- Perimeter

### Problem Solving Strategies

- Diagram/Key
- Model
- Formula
- Equations
- Scale drawing

### Formal Mathematical Language and Symbolic Notation

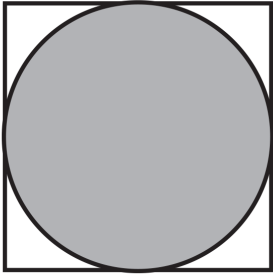
- Diagram
- Model
- Table
- Formula
- Equation
- Circle
- Square
- Area
- Perimeter
- Maximum
- Maximize
- Inscribed
- Area of a circle ( $A = \pi r^2$ )
- Radius
- Diameter
- Area of a square ( $A = S^2$ )
- Length
- Half
- Quarter
- Pi ( $\pi$ )
- Approximation
- Round
- Foot ('), square foot (sq. ft.)

### Possible Solution(s)

The stake must be placed in the center of the yard otherwise the goat will be able to jump the fence. The rope should be 75 feet long so the goat has about 17,662.5 sq. ft. to eat grass. 4,837.5 sq. ft. of grass will still need to be mowed. It would take 1.6 hours to cut the remainder of the grass.

The square yard has a perimeter (fence) of 600 feet. Because all four sides in the square are equal, this means a side of the yard must be  $600 \div 4 = 150$  feet.

The goat will be staked in the center of the yard, making the rope the radius of the circle (using 3.14 to approximate pi):



Radius of circle =  $\frac{1}{2}$  side of square

Radius of circle = 75 feet

Grazing area = whole circle area

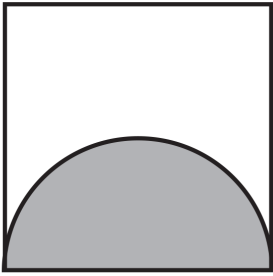
Grazing area =  $\pi r^2$

Grazing area  $\approx 3.14 \times 75^2$

Grazing area  $\approx 17,662.5 \text{ ft}^2$

The area left to be mowed is  $(150)^2 - 17,662.5 = 4,837.5$  square feet.

Other possible placements that may be explored include:



Radius of circle =  $\frac{1}{2}$  of side of square

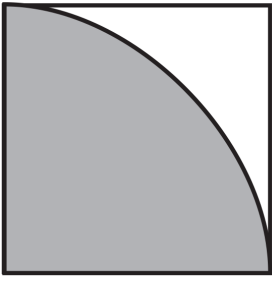
Radius of circle = 75 feet

Grazing area =  $\frac{1}{2}$  of the circle area

Grazing area =  $\frac{1}{2} (\pi r^2)$

Grazing area  $\approx \frac{1}{2} (3.14 \times 75^2)$

Grazing area  $\approx 8,831.25 \text{ ft}^2$



Radius of circle = side of square

Radius of circle = 150 feet

Grazing area =  $\frac{1}{4}$  of the circle area

Grazing area =  $\frac{1}{4} (\pi r^2)$

Grazing area  $\approx \frac{1}{4} (3.14 \times 150^2)$

Grazing area  $\approx 17,662.5 \text{ ft}^2$

### Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- When a circle is inscribed in a square, the center of the square is the same point as the center of the circle.
- The length of the diameter of a circle inscribed in a square is equal to length of the side of the square.
- The grazing area is circular because all points are an equal distance (the rope length) from the center point (the stake) to the edge of the square yard, which is the definition of a circle.
- The area covered by the corner placement of the stake in the square field has the same area as the center placement of the stake.
- The radius of the corner placement of the stake is twice (150) the length of the center placement of the stake (75).
- The area created by the corner placement of the stake is a quarter of a circle, so I have to adjust for area formula of a circle to  $\frac{1}{4}\pi r^2 \approx 17,662.5$ .
- The area created by the center placement of the stake is a full circle so I can use the formula  $\pi r^2 \approx 17,662.5$ .
- If the rope reached to the corner of the square yard, instead of the side, this would make a circumscribed circle (where the goat could graze the entire yard) instead of the inscribed circle (completely contained in the yard).
- Solve more than one way to verify the answer.