Exemplars

Title: Billy's Goat
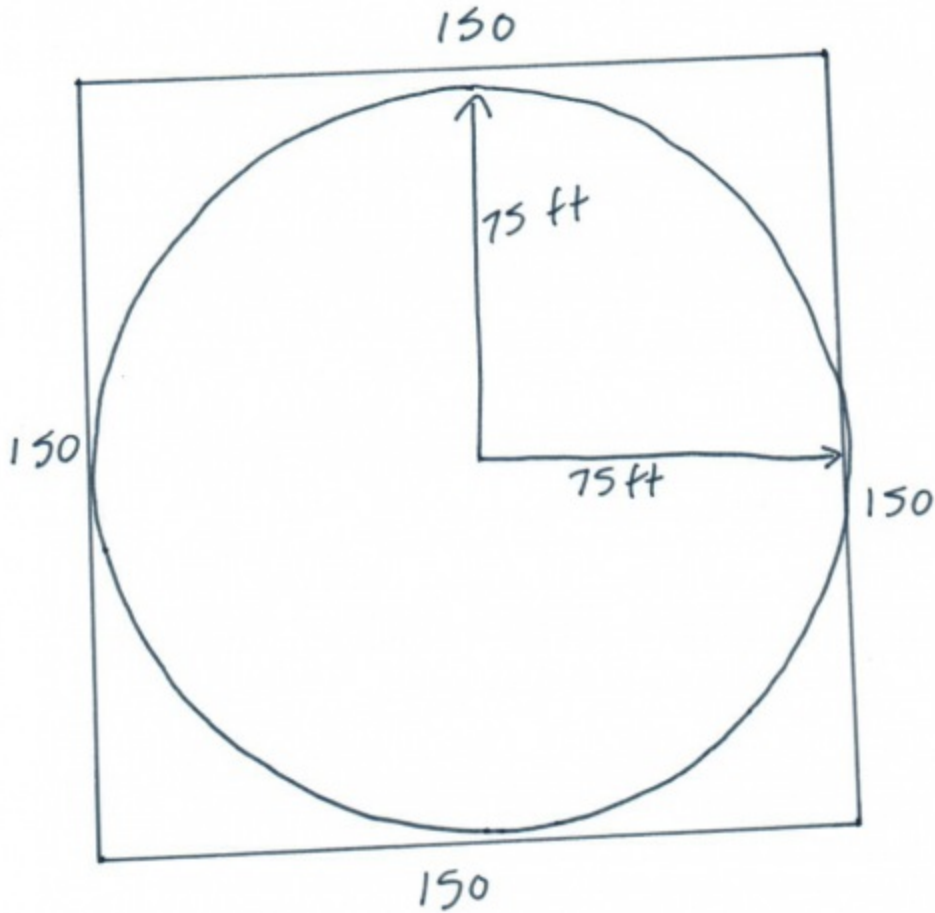
Achievement Level: Novice 1

Criteria and Performance Level	Rationales
Problem Solving *Apprentice*	The student shows minimum engagement in the task. No strategy is presented towards reaching a solution to the task.
Reasoning & Proof *Novice*	No mathematical justification or reasoning is provided to support the limited information present.
Communication *Novice*	No purpose is communicated for the task. No formal mathematical terms are included. No strategy is shared for how the student will solve the task.
Connections *Novice*	No connections are attempted by the student.
Representation *Apprentice*	The student constructs the beginning of a strong representation. The diagram lacks labels for much of the information provided.

Exemplars

Achievement Level: Novice 1

150

75 ft

150

75 ft

150

150

Title: Billy's Goat

Achievement Level: Apprentice 1

Criteria and Performance Level	Rationales
Problem Solving *Apprentice*	A partially correct strategy is utilized to correctly answer several of the important questions in the task. The student uses a successful strategy to find where to place the stack, the length of the rope, how much space the goat will be able to graze, and finally, the area of the square. A problem is that the student decides to incorrectly "divide the area of the circle by it (17671.458) and got 1.273 left to mow". The student also fails to answer how much time this is going to require to mow.
Reasoning & Proof *Apprentice*	In the beginning, the student provides some mathematical basis for their conclusions. Some correct reasoning is present in the beginning. In the last stage provided, the student fails to define their reasoning for dividing the area of the circle by the area of the square and reverses the numbers they are working with.
Communication *Apprentice*	The student clearly defines the purpose for the problem. The student provides a clear and sequenced approach for solving the section of the problem they completed. Some challenges within the work center on the lack of labels for many of the values provided, "divide 150 (what) by 2. I got 75 (what)". The area of the circle and square lack labels.
Connections *Practitioner*	The student attempts to explore an interesting mathematical connection by asking a question: "what would happen if the backyard was rectangle."
Representation *Apprentice*	The student attempts to construct a representation but it lacks significant labels and clarifying information.

Achievement Level: Apprentice 1

P/S	R/P	Com	Con	Rep	A/Level
A	A	A	P	A	A

Billys Goat Problem

The problem was, we had to find a place a in a square yard with a perimeter of 600 where billy the goat could not jump over the fence but could get the most grass out of the area while he was tied to a stake. So we needed to find out how long the rope would need to be, where the stake should be placed, how much space the goat will have to graze, and how much grass will be left to mow.

The first thing i did to solve Billys goat problem was to figure out where to place the stake. I decided the best spot in the yard that corresponded with the other variables was in the **middle** (75 feet away from each side). Then I needed to figure out how long the rope would need to be. The yard is 150 feet long and I placed the stake in the middle so I decided to divide 150 by 2. I got 75 for the ropes length. Next I needed to figure out how much space Billy the goat will have to graze. The radius of each side was 75 so I multiplied 75x75x pie which equaled 17671.458 space for the goat to graze. Finally I needed to figure out How much grass would be left to mow. I figured out the area of the square by multiplying length x width and got 22500. Then I divide the area of the circle by it (17671.458) and got **1,273 grass left to mow**.

One thing I wondered after doing thind problem is what would happen if the backyard was rectangle.

Overall this problem taught me to trust myself and not shoot down my ideas so much. Overall I think I got proficient on Billy's Goat Problem.

Title: Billy's Goat

Achievement Level: Apprentice 2

Criteria and Performance Level	Rationales
Problem Solving *Practitioner*	Based on the student's mathematics, it is evident that the student understands the underlying mathematics of the problem. However, little attempt is made to clearly justify or explain their reasoning. Overall, the student chooses strategies which lead to a correct solution of 69.66 minutes. Prior knowledge is utilized throughout the problem, "1 foot2 = 144 in2", "πr2 * 3.14 = area of circle".
Reasoning & Proof *Apprentice*	Did the student provide adequate mathematical basis within their solution argument? Where does 150 come from? What does it represent? Where does "A = 22500 ft2" come from? Where does the 75 in the equation 752 * 3.14 = 17662.5 ft2 come from? What does 17662.5 ft2 represent? A significant portion of the student's work requires us to infer the student's meaning.
Communication *Apprentice*	The student fails to provide a clear, organized explanation for how they solved this problem. Significant interpretation is required concerning what many of the numbers, variables or calculations actually mean. Stages of the solution process and numerous variable are utilized which are not defined or explained. The student also fails to define the purpose of the solution.
Connections *Novice*	No connection is provided within the student's solution.

| Representation *Apprentice*	An attempt is made to construct a representation but it lacks labels for the numbers provided and any labels defining what each element of the drawing represent.

Note:

The overall achievement level for this piece of student work falls under Exemplars exception to the rule category. If a student has all Apprentice scores or above, but a Novice in "Connections," the student may still receive an achievement level score of Apprentice. To learn more about Exemplars scoring, please refer to the section of your dashboard called "Tools for Success" and click on the link for "Using the Assessment Rubric."

Achievement Level: Apprentice 2

Billy's Goat

P/S	R/P	Com	Con	Rep	A/Level
P	A	A	N	A	A

150

75

150
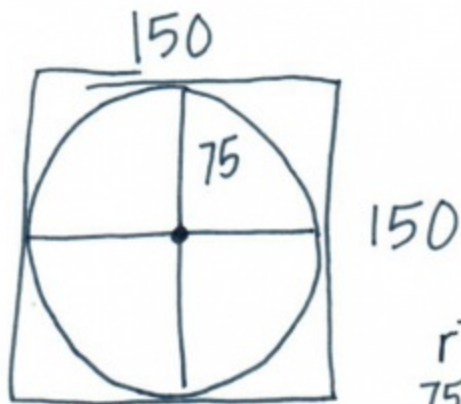
$P = 600$ ft
$SL = 150$ ft
$A = 22500$ ft^2

Rope 75 ft long

$r^2 \cdot 3.14 = $ area of cirle
$75^2 \cdot 3.14 = 17662.5$ ft^2

$22500 - 17662.5 = 4837.5$ ft^2

I can convert 4837.5 square feet into 696,600 inches squared

4837.5 ft^2

1 foot^2 = 144 in^2

$144 \cdot 4837.5 = 696600$ inches^2

1 hour 9 minutes 1 second

$60/100 = .6$ 10,000 inches p minute

$696600/10,000 = $
69.66

Note: pi rounded to 3.14

69.66 minutes

Title: Billy's Goat

Achievement Level: Apprentice 3

Criteria and Performance Level	Rationales

Note:

Rationale tbd

① Billy needs to find the best place to put his goat so he will have the Most grass but Can not jump the fence.

② I am going to draw a square with Side Length of 150 feet so I have 600 feet as a total perimeter. You make a circle start in the center with ha diemeter of 75 feet then find the area of the circle and divied by the area of the square.

75
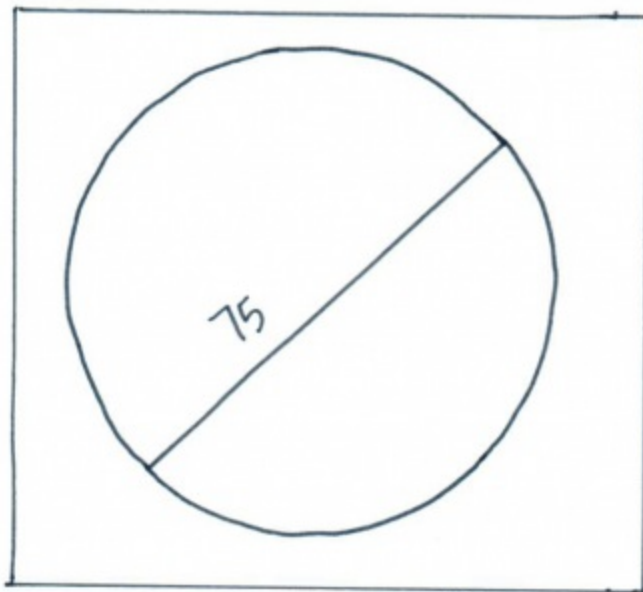
$75 \times \pi = 21.99$ Feet $= A$

$150 \times 150 = 22500 = A$

Title: Billy's Goat

Achievement Level: Practitioner 1

Criteria and Performance Level	Rationales
Problem Solving *Practitioner*	The student uses correct strategies throughout the problem to reach a correct solution. The student correctly applies prior knowledge when finding the area of the circle, "πR2" , and when defining the square yard as having 4 equal sides.
Reasoning & Proof *Practitioner*	The student constructs a systematic approach to provide mathematical reasoning for their final conclusion. The numbered steps with calculations helps clarify their mathematical justification.
Communication *Practitioner*	The student creates a clear, methodical, and well organized explanation of how they reached their final solution. Appropriate vocabulary is used throughout to clarify the students solution strategy. The numbered steps helps to define the stages of the students solution pathway.
Connections *Practitioner*	The student clarifies the situational context of the task that the rope tied to the goat will allow for the goat to eat in a circle around the stack. The student also explores the broader context of the task by suggesting that an alternative solution is "an extension on top of the fence."
Representation *Expert*	The student produces 2 separate representations that 1) help to analyze the relationship between square feet and square inches and 2) clarify the situation of the goat's grazing area within the area of the square fence.

Achievement Level: Practitioner 1

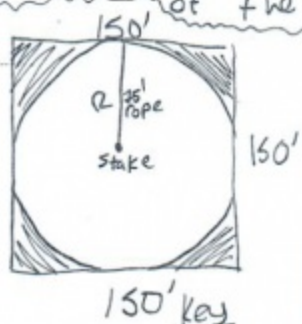| P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	E	P

Summary
How can we give the goat the most space without allowing it to jump the fence? And, how much space is left ungrazed and how much time to mow it?

Connection
It would probably be more time saving and efficient to either put a pen inside of the fence or build an extention on top of the fence.

150'

R 75' rope
stake
150'

150' key
□ = area grazed
▨ = area to mow

① Perimeter of Yard
600ft total
÷ 4 sides
= 150' per side

② Length of Rope
150' across whole yard, and stake is in the middle to allow grazing so the longest the rope can be without jumping the fence is 75! This allows max grazing area

③ Area of grazing
πR^2 = Area
Radius of the Circle the goat can graze is 75ft because of the rope, so to calculate the area the goat can graze it would be $\pi 75^2$ which equals 17,671.46ft² to graze.

④ Area to be mowed
The yard is 150' x 150' which is a total area of 22500ft². 17,671.46ft² are used by the goat for grazing.
22,500 - 17,671.46 =
4,828.54ft² left to mow

$1ft^2 = 144in^2$

* 1ft ↕12in 144in²
12in
1ft
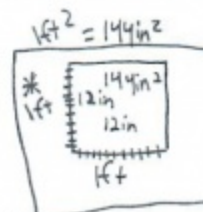
⑤ The mower cuts 15,000in² in 1½ min. There is 4,828.54ft² left to mow. Each ft² = 144in² * 4,828.54ft²
X/44 = 645,300.76 in² to mow.

⑥ 695,309.76 in² to mow
÷ 15,000in² per 1½ min
= 46.35 x 1½ min =
69.5 min =
1h 9min 30 seconds
to mow the remaining grass

Answers
stake in center of yard
75' rope
1H 9m 30s to mow grass left over
695,309.76 ² of grass to mow

Title: Billy's Goat

Achievement Level: Practitioner 2

Criteria and Performance Level	Rationales
Problem Solving *Practitioner*	The student utilizes a correct strategy to reach the correct answer for the problem. Prior knowledge is used to find the area of the circle "$\pi r2$" and to find the area of the fenced in area, "Area Square = length x width".
Reasoning & Proof *Practitioner*	The student constructs a systematic, mathematically based argument for each stage of their solution process. Numbering the stages helps clarify for the audience the sequence the student utilized.
Communication *Practitioner*	The student defines the purpose of the problem by stating the problem in their own words. The student provides a clear, sequenced solution approach which is easy to follow towards their final conclusion. No interpretation of their strategy is required. Formal math language is utilized including, area, circle, center, square.
Connections *Practitioner*	The student clarifies the situational context that the "goat can eat the area of a circle" around the stack. This connection becomes the foundation for the student's solution to this task.
Representation *Practitioner*	The student constructs a diagram to portray the relationship between the area of the total fenced in area, the area of the circle the goat can eat, and the area which will need to be mowed.

Achievement Level: Practitioner 2

I am trying to figure out...how much grass the goat will leave the kids to mow and how much time it will take them to mow it.

To do this, I will... subtract the area of the circle of grass the goat is in from the area of the yard.
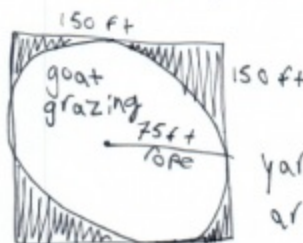
① If I stake the goat 75 ft from all sides (in the center of the square fence) the goat can be in a circle.

The area of a circle is $\pi \cdot radius^2$

$$area = \pi \cdot 75^2$$

$$goat's \ circle = 17,671.5 \ ft^2$$

② Square fence perimeter is 600 ft total

$$600 \ ft \div 4 = 150 \ ft$$

③ Area left to mow

$$22500 \ ft^2 - 17671.5 \ ft^2 =$$

↑ Square area   ↑ Circle area $\boxed{4828.5 \ ft^2}$

yard area = length × width

$$area = 150 \ ft \times 150 \ ft$$

$$area = 22500 \ ft^2$$

④ Time it takes to mow

first, I have to go from square feet to square inches

$$4828.5 \ ft \times 144 \ in^2 = 695,304 \ in^2 \leftarrow$$ because there 144 in² in one square foot

Divide area to mow by mowing rate

$$695,304 \ in^2 \div 15,000 \ in^2 \ per \ 1\tfrac{1}{2} \ minute = 46.35 \ (1\tfrac{1}{2} \ minutes)$$

$$46.35 \times 1\tfrac{1}{2} \ min = 69.5 \ minutes$$

$\boxed{1 \ hour, 9 \ minutes, and \ 30 \ seconds \ to \ mow \ remaining \ lawn}$

Title: Billy's Goat

Achievement Level: Practitioner 3

Criteria and Performance Level	Rationales
Problem Solving *Practitioner*	The student uses a correct strategy and provides a correct answer to the task. The student applies prior knowledge in finding the area of the circle "pi • r2 ".
Reasoning & Proof *Practitioner*	The student constructs a systematic, mathematical justification with significant information supporting their final solution. The student provides the adequate mathematical calculations for each stage of their solution. Feedback to student may include a suggestion to add additional mathematical calculations specifically how they determined "the total area of Billy's yard, 22,500 ft2". Converting 15000 to 1.5 minutes to an equivalent rate of 10000 to 1 minute simplified the resulting calculations considerably.
Communication *Expert*	The student clearly identifies the purpose of the task by stating the questions to be answered along their solution path. Their approach is methodical and coherent. The student provides terrific insight along the way towards their thinking process, "Question 1 isn't even a mathematical question", and "where question 3 was simple, Question 4 is quite the opposite," finally "the purpose of that is just so I have easier numbers to work with later on".
Connections *Expert*	The student explains their rationale for why the calculation for the time needed to mow the lawn would not likely be realistic. This generalization into other cases is evidence of an expert.

Representation	The student constructs an appropriate visual which portrays the relationship between the area of the fenced in area, the area of the
Practitioner	circle and the area that will need to be mowed.

Achievement Level: Practitioner 3

P/S	R/P	Com	Con	Rep	A/Level
P	P	E	E	P	P

Billy's Goat

Question 1 | Where should the stake be placed so that the goat will have maximum grazing area?

Question 1 isn't even a mathematical question. Logically, for the goat to have maximum potential grazing area, the stake would go in the center of the yard. The goat, being roped to the stake, would eat in a circle in the center of the square shaped yard

Question 2 | What length of rope should he use? and how much space will the goat have to graze?

Question 2 is divided into 2 parts which I'll call 2a and 2b. Like I previously said, the goat will eat in a circular shape if tethered to the stake in the center of the yard. To give the goat maximum grazing area, the rope should be long enough to reach the edge of the yard from the center of it. Due to information from the story, we know that Billy's yard is 150ft x 150ft. That means that if the rope has to be half of that, the rope length should be 75 ft.

From there, we move on to 2b. As the goat eats in a circle, we have to find the area of that. We can use 75 as the radius of our circle. From there I simply used the $pi \cdot r^2$ equation, (pi rounded to 3.14). I got an area of 17662.5 ft².

connection | I think that not all of the answers, in specific the last one, can be perfect numbers. The lawn mower will not be able to reliably mow all 696,600 square inches of grass in precisely 1 hour, 9 minutes and 1 second.

Exemplars

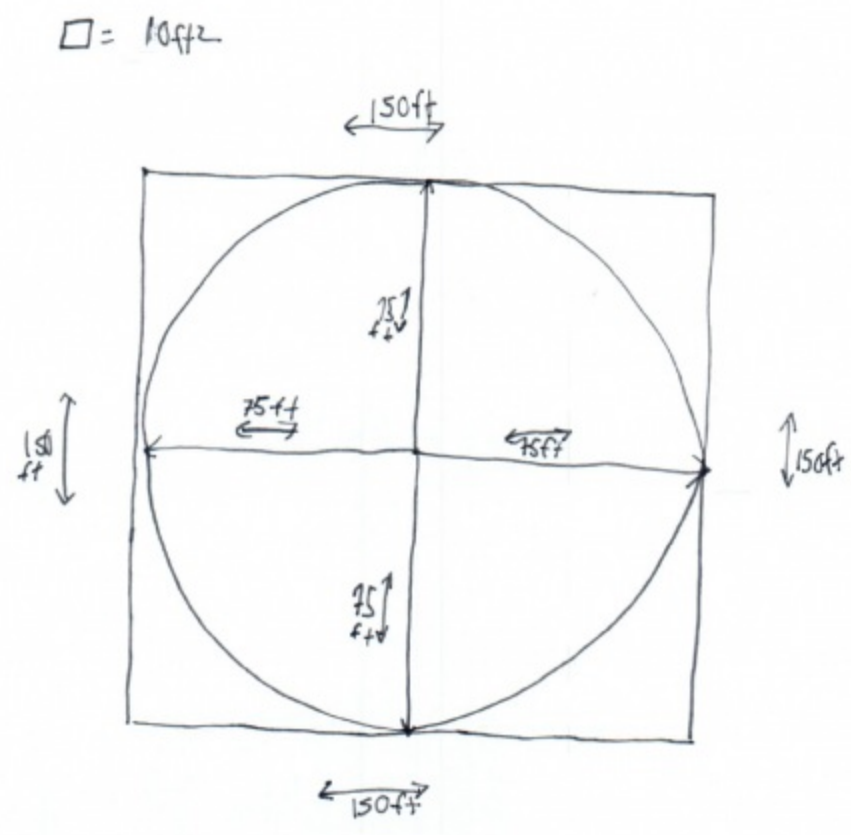|	
Question 3	How much grass would be left for the kids to mow?
	Question 3 is another very simple one. To get the answer, I subtracted the area of the circle, 17662.5 ft² from the total area of Billy's yard, 22,500 ft². The grass for the boys to mow is 4837.5 ft².
Question 4	If the lawn mower cuts 15,000 square inches in 1.5 minutes, about how many more hours are needed to cut the remainder of the grass.
	Where Question 3 was simple, Question 4 is quite the opposite. I started by recognizing that if the rate of square inches to time is 15000 to 1.5 minutes. I can chane that to 10000 to 1 minute. The purpose of that is just so I have easier numbers to work with later on. Next, I have to make the units the same. To convert square feet to square inches, I multiplied the square feet by 144. 144·4837.5 = 696,600 inches. From there, I divided 696,600 by 10,000. Remember when I said I wanted easier numbers? Anyway. I got 69.66, or 69 minutes. As for the .66, I called that ⅔. That means that .6 is equivlent to 36 seconds. I rounded down the .06. My final answer was 1 hour, 9 minutes, and 36 seconds.
Connection Continued	When the lawn mower cuts corners, or backs up, time will be spent NOT mowing the lawn. Also, the lawn mower could get stuck, or broken in some other way that would take more time. My point is that the mower cannot realistically cut the grass at the exact same time reliably.

☐ = 10ft²

150ft

75 ft²

75ft

150 ft

75ft

150ft

75 ft²

150ft

Title: Billy's Goat

Achievement Level: Expert 1

Criteria and Performance Level	Rationales
Problem Solving *Expert*	The student utilizes an effective strategy to answer the problem. Along the way, the student analyzes additional questions relevant to a highly detailed solution to the problem, ie. considering the length of rope needed to tie around the stack and the goats neck. The student also considers alternative strategies of simply building the fence higher, and brings in outside information to consider the cost of the fence.
Reasoning & Proof *Expert*	The student provides rigorous mathematical arguments for their calculations of their final answers. Accurate, mathematical evidence is used throughout to justify their conclusions.
Communication *Expert*	The student provides a coherent, sequenced approach for solving the problem. The student achieves an expert through the insights they provide about their reasoning in several places in the task which go beyond the original task, including their discussion of the additional rope needed to tie the goat, the idea of using Pythagorean theorem to calculate the length of the rope to the fence plus the considering the height of the goat, and discussing an analysis of an alternative solution of simply building a higher fence and weighing this against the use of the rope.
Connections *Expert*	The student goes beyond the original problem to investigate additional questions relating to the problem. Extending the solution to questions beyond the original task demonstrates expert thinking.

Representation *Expert*	The student constructs two mathematical representations to portray their thinking and help with developing an effective solution for solving the problem. The image of the goat with the 2ft label helps to clarify the discussion of using Pythagorean Theorem in their discussion for finding the total length of the rope.

Exemplars

Achievement Level: Expert 1

In the problem Billy's Goat, Billy's family owns a goat that likes to jump their fence. Billy is trying to find a way to tie the goat so that it can have the most possible grazing space without jumping the fence. Their yard is square and the total fence length is 600 feet. Where should we place the stake for the maximum grazing room, how long should the rope be, how much grazing room will the goat have, how much grass would be left, and how long would it take to mow the remaining grass if it takes 1.5 minutes to mow 15,000 square inches of grass?

The stake should be placed exactly in the middle, because the rope can only go as far as the distance to the closest point on the fence, or else the goat will jump over. If you place the stake in the exact middle, then the distance to the fence is as far as possible in every direction, giving the maximum area for grazing.

One would assume that since the distance from the center of the yard to the fence is 75 feet, then the rope would be 75 feet as well. That's a good approximation, and it shows us about how long the rope will be, but if you want to get more precise you have to factor in the length of rope needed to tie the rope securely around the goat's neck and around the stake.

If we assume that the stake is 2 inches in diameter, then we can find the circumference. The radius would then be 1. 1^2 is 1, of course, and $1*\pi$ gives us about 3.14 inches. Add in about 8in to tie the rope securely, and we get about 11.14 inches for the stake.

The goat's neck is more tricky to calculate, owing to the fact that it's not circular. When measured on an image, keeping in mind that an average goat is about 2 feet tall, the neck is about 9.6 inches long and 4 inches wide. When you calculate the circumference of the neck, you get 23.103 inches. Another 8 inches is used to tie the rope securely. 23.103+8 is 31.103. We add together 11.14" and 31.103" to get the length for the entire tied portion of the rope, giving us 42.243".

The rest of the rope is simple to calculate. Assuming that the rope is set against the ground where it's tied around the stake, we can use the Pythagorean Theorem to find the length of the rope. $75^2+2^2=5629$, and $\sqrt{5629}$ is 75.027. When you translate the decimal into inches, you get about 75 feet and 3 inches.

When adding the rope used to tie, you have to keep in mind that the units we were using when calculating were inches, not feet. Once you convert to inches by dividing by 12, we do 42.243" ÷ 12, you get about 3 feet, 6.24 inches. Add together the straight rope and the tied part, and you end up with a sum of 78' 9.24" of rope.
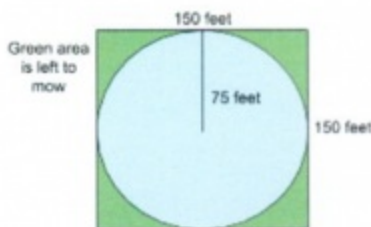
How much grass does this give the goat to graze? First, we know that the radius of the circle the goat can reach, 75 feet. This means we can find the area of the circle using πr^2. 75 has to be squared, which is 5625*3.14=17662.5 feet squared for the goat to eat.

Next, we want to figure out how much is left to mow. Given that the total fence length is 600 feet and their yard is square, each side is 150 feet long. This means that the

area of the whole yard is 150*150 feet, or 22500 ft^2 big. The goat eats some of this, so to find the amount of grass needed to mow, we find the difference between 22500 and 17662.5. 22500-17662.5=4837.5 ft^2 left.

Assuming a constant rate of mowing (15,000 square inches every 1.5 minutes) then we can calculate how long it would take to mow the leftover grass. First you have to turn the feet into square inches. This isn't simply multiplying by 12 this time, since we're working with square inches, which means we have to multiply by 144 instead. That gives us 696600 square inches (4837.5 ft^2 x 144 in^2 / ft^2). We can turn these into 1.5 minute chunks by dividing by 15,000, leaving us with 46.44 chunks of 1.5 minutes. 46.44*1.5 turns it into single minutes and leaves us with 69.66 minutes to mow the remaining grass. Since there's 60 minutes to an hour and 0.66 rounds almost exactly to ⅔, we can say that it would take 1 hour, 9 minutes and 40 seconds to mow the lawn.

Of course, using a rope isn't the only way of keeping the goat in while letting it have the most grazing space. Another, much simpler is to build a taller fence that it can't jump over.

The question is when would the fence become more cost efficient? Considering the approximate $24/foot cost for a wooden fence, it would cost $14,400 to make a fence all the way around their lawn. However, you do have to pay for gas with a mower. If the mower uses about 1.25 gallons an hour, that's around 1.46 gallons of gas for the lawn with the rope method. If gas costs around $2.50 a gallon, it would cost $3.65 every time you mow the lawn. 14,400 divided by 3.65 is 3945.21 mowings of the lawn until the fence is less expensive.

All in all, you're probably better off with the rope.